NONLINEAR DYNAMIC ANALYSIS OF 2-D REINFORCED CONCRETE BUILDING STRUCTURES

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SYNOPSIS

This is a state-of-the-art paper about nonlinear dynamic analysis of plane reinforced concrete building structures through consistent prejudices of a single person. The paper reviews the behaviour of reinforced concrete members and their subassemblages observed during laboratory tests. Then, different hysteresis and analytical models of reinforced concrete members are studied. Finally, their application to the simulation of behaviour of small-scale plane building models observed on earthquake simulators is discussed.

RESUME

On fait état des connaissances sur l'analyse non-linéaire du comportement dynamique des bâtisses construites en béton armé. Des résultats en laboratoire des membrures et assemblages en béton armé sont présentés. Une énumération des techniques existantes de modèles non-linéaires et de dissipation d'énergie utilisés en béton armé est préparée. L'application est faite à des modèles réduits en laboratoire.

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INTRODUCTION

Dynamic response of a structure can be caused by different loading conditions such as, (a) earthquake ground motion, (b) wind pressure, (c) wave action, (d) blast, (e) machine vibration, and (f) traffic movement. Among these, inelastic response is mainly caused by earthquake motions and accidental blasts. Consequently, more research on nonlinear structural behaviour has been carried out in relation to earthquake problems. This paper describes the research development in earthquake engineering.

The dynamic behaviour of a structure might appear to be best studied through a series of dynamic tests of real structures. Various testing methods are available. However, dynamic characteristics up to failure cannot be identified solely through a dynamic test of a real structure by the following reasons: (a) it is difficult to analyze data because the response includes complex interactions of various parameters within a real building; (b) it is very expensive to build a structure, as a specimen, for destructive testing; and (c) the capacity of a shaker is insufficient to cause failure of a full-scale structure.

Consequently, dynamic tests of real buildings are rather aimed toward obtaining data, (a) to confirm the validity of mathematical modelling techniques for a linearly elastic structure, and (b) to obtain damping characteristics of different types of structures. A specifically designed laboratory test becomes inevitable to complement the weakness of full-scale tests and to study the effect of individual parameters.

Dynamic problems are different from static ones in the following respects: (a) inertia force, (b) damping, (c) strain rate (velocity), and (d) stress reversals due to oscillation. The inertia force is a product of absolute acceleration and mass. In a building structure, mass is normally assumed to concentrate at a floor level. Mass distribution within each member needs be included when local vibration of the member is of interest. Damping characteristics, strain rate effect and the effect of stress reversals are discussed further.

Nonlinear dynamic analysis of a reinforced concrete structure requires two types of mathematic modelling: (a) a model to represent the distribution of stiffness along a member, and (b) a model to represent the force-deformation relationship under stress reversals. A variety of models for plane reinforced concrete building structures are reviewed in this paper.

DAMPING

Any mechanical system possesses some energy dissipating mechanisms; for example, (a) inelastic hysteretic energy dissipation, (b) radiation of kinetic energy through foundation, (c) kinetic friction, (d) viscosity in materials, and (e) aerodynamic effects. However, the state-of-the-art does not provide a method to determine the damping capacity based on the material properties and geometrical characteristics of a structure. Such energy dissipation, vaguely termed as "damping", is most often assumed to be of viscous type because of its mathematical simplicity. In the nonlinear dynamic analysis of a structure, a damping matrix is normally assumed to be proportional to the constant mass matrix and/or the instantaneous stiffness matrix. The damping is known to have a major effect on the dynamic response amplitude of a linearly elastic structural model.

Damping capacity is often determined during a sinusoidal steady-state resonant test by the band-width of the response curve. Figure 1 shows acceleration response amplitudes during a series of steady-state tests of a reinforced concrete building at different intensity excitation levels (1). Measured damping capacity varied from 0.6 per cent of critical in a man-excited test to a maximum of nearly 2 per cent. Note that damping capacity is not a unique value of the structure, but it depends on the level of excitation. Further investigation is necessary to determine the energy dissipation characteristics of a structure.

STRAIN RATE EFFECT

The response of a structure during an earthquake is dynamic. The stiffness and strength of various materials are known to increase with the rate of loading. However, it is technically more difficult to test structural elements under a realistic dynamic condition in a laboratory.

The effect of strain rate on the behaviour of the reinforced concrete was reported by Mahin and Bertero (2), who tested four medium-size reinforced concrete beams under third-point loads at constant speeds during loading and unloading. Important findings from this investigation are as follows: (a) displacement rates showed practically little effect on initial stiffness; (b) high strain rates increased the yield resistance by more than 20 per cent; but (c) high strain rates caused small differences in either stiffness or resistance in subsequent cycles at the same displacement amplitudes; (d) strain rate effect on resistance diminished with increased deformation in a strain hardening range; and (e) no substantial changes were observed in ductility and overall energy absorption capacity.

It is important to recognize that the strain rate (velocity) during an oscillation is highest at low stress level, and that the rate gradually decreases toward a peak strain. Cracking and yielding of a reinforced concrete member reduce the stiffness, hence the period of oscillation becomes longer with structural damage. Furthermore, such damage is normally caused by first several modes of vibration, whose periods are relatively long. Therefore, the strain rate effect may not be as important as the material tests under extraordinary high and constant strain rates indicate.

Indeed, dynamic hysteresis loops obtained from one-storey one-bay reinforced concrete frames were compared favourably with those obtained from quasi-static tests (3).

Therefore, the hysteretic behaviour of reinforced concrete members observed during a "static" test can be utilized in a nonlinear dynamic analysis of reinforced concrete structures. When a structure is studied under a blast load, the effect of strain rate need be considered.

BEHAVIOUR OF REINFORCED CONCRETE MEMBERS

Behaviour of reinforced concrete materials under full range of stress reversals and under general multi-stress states has not been completely understood to a stage where analytical material models can predict every aspect of inelastic member behaviour. Although such models are useful for qualitative and quantitative examination of a critical region of a member, it is not feasible nor recommended to apply these material models to an analysis of an entire building structure. Normally the computing effort required for an analysis using material models becomes extremely expensive compared to the gain in accuracy and reliability of the computed results. Therefore, it becomes more important to study the behaviour of reinforced concrete isolated members (beams, columns, slabs and walls) and their sub-assemblies (beam-column, slab-column and slab-wall connections) so that their analytical models can be developed for use in the response analysis of a complete structure.

Thousands of reinforced concrete members have been tested under static load reversals. A typical force-deflection curve of a cantilever column (305 x 305 x 1524-mm) under lateral load reversals is shown in Fig. 2 (4). Note the following observations: (a) tensile cracking of concrete and yielding of longitudinal reinforcement reduced the stiffness; (b) when a deflection reversal is repeated at the same newly attained maximum amplitude (for example, cycles 3 and 4) the loading stiffness in the second cycle is lower than that in the first cycle, although the resistances at the peak displacement are almost identical; (c) average stiffness (peak-to-peak) of a complete cycle decreases with a previous maximum displacement amplitude. For example, the peak-to-peak stiffness of cycle 5, after large amplitude displacement reversals, is significantly reduced from that of cycle 2, although subjected to comparable displacement amplitude reversals. Therefore, it can be seen that the hysteretic behaviour of the reinforced concrete is sensitive to loading history.

Flexural Characteristics Under Reversed Loading

A regular reinforced concrete member resists bending and shear. Let us consider deformations corresponding to bending and shear. Flexural deformation "index", or average curvature, is obtained from longitudinal strain measurements at two levels, assuming that a plane section remains plane during deformation. This flexural deformation index does not represent the flexural deformation in a strict sense because a plane section does not remain plane in a region where an extensive shear deformation occurs. However, the index is useful to understand flexural deformation characteristics qualitatively.

A typical moment-flexural deformation index curve obtained from a simply supported beam test (5) is shown in Fig. 3. It is interesting to note that the stiffness during loading gradually decreases with load, forming a fat hysteresis loop, and absorbing a large amount of hysteretic energy. The hysteresis loops remain almost identical even after several load reversals at the same displacement amplitude beyond yielding. Consequently, vibration energy of a structure can be dissipated through flexural hysteresis loops without causing the reduction in the resistance. Therefore, current design provisions encourage the reinforced concrete member to be designed to behave dominantly in flexure. Many hysteretic models, as discussed later, are currently available to represent the flexural behaviour of reinforced concrete members.

The increase in axial force decreases the ductility of a reinforced concrete member, but increases force levels corresponding to, (a) tensile cracking of concrete, (b) tensile yielding of longitudinal reinforcement, and (c) inclined shear cracking of concrete.

Shear Deformation Characteristics Under Load Reversals

Similar to the flexural deformation index, a shear deformation index is defined from strain measurements in the two diagonal directions to study qualitatively the shear deformation characteristics. Again, this index does not represent the true shear deformation because the interference of shear and flexural deformations exists.

A typical lateral load-shear deformation index curve obtained from a simply supported beam test (5) is shown in Fig. 4. Contrary to the flexural stiffness, the stiffness during loading gradually increases with load, exhibiting a "pinching" in the curve. The hysteretic energy dissipation is smaller. The hysteresis loop decays with number of load reversals, resulting in a smaller resistance at the same peak displacement in each repeated loading cycle. The effect of loading history is pronounced on the shear deformation.

Although the curve shows a "yielding" phenomenon, it is important to recognize that the shear force of the member was limited by flexural yielding at the critical section rather than by "yielding" in shear. This "yielding" clearly indicates the interaction of shear and bending moment.

The "pinching" in the force-deformation curve is obviously less desirable. The shear span to effective depth ratio is the most significant parameter that controls the shear deformation. Decreasing the shear span to depth ratio causes a more pronounced "pinching" in the curve, and a faster degradation of hysteretic energy dissipating capacity.

Considerable improvements in delaying and reducing the degrading effects can be accomplished by using closely spaced ties. Existence of axial force tends to retard the decrease in stiffness and strength with cycles because the cracked concrete surfaces are pressed together by the axial force (6).

However, it is hard to eliminate this undesirable effect when high shear stress is present. Consequently, it becomes important to include this degrading behaviour in a behavioural model for a short, deep reinforced concrete member. Unfortunately, the current state of the knowledge is not sufficient to define the stiffness degrading parameters on the basis of the geometry of a member and the material properties.

Bar Slip and Bond Deterioration

When a structural element is framed into another element, some deformation is initiated within the other element. Consider a beam-column sub-assembly. Bertero and Popov (7) reported a significant rotation at a beam end caused by the slippage (pull-out) of the beams' main longitudinal reinforcement within the beam-column joint (Fig. 5). The general shape of the moment-bar slip rotation curve is similar to that of the shear force - shear deformation index curve shown in Fig. 4, demonstrating a pronounced pinching of a hysteresis loop. The contribution of bar slip to total deformation cannot be neglected, especially in a stiff member (short or deep) where the deformation of a member is small.

Summary

Behaviour of a reinforced concrete member and sub-assemblies under static reversed cyclic loading is reviewed briefly. These observations are applicable to beams, columns, walls and their sub-assemblies. Although brittle shear, compression, and anchorage failures of the reinforced concrete are normally prevented through the application of building code provisions, structural members and sub-assemblies can not be completely free from those deformations. It is desirable that an analytical model of a reinforced concrete member should include these characteristics attributable to flexure, shear, axial force and bar slip.

HYSTERETIC MODELS FOR REINFORCED CONCRETE

A hysteresis model must be able to provide the stiffness and resistance under any displacement history. At the same time, the basic characteristics need be defined by the member geometry and material properties. The current state of knowledge on the reinforced concrete behaviour is sufficient to determine the shape of force-flexural deformation hysteresis from given member geometry and material properties. However, it is not sufficient to determine the degree of stiffness degradation due to the deterioration of shear resisting mechanisms and due to bar slip under load reversals.

It is interesting to realize the inelastic dynamic analysis methods were developed by purely analytical people in the late 1950's with the advent of digital computers, and remained in their hands through the 1960's. When the numerical methods for nonlinear dynamic analyses of a simple structure were made easily accessible to experimental investigators in the late 1960's, more realistic hysteresis models were developed by many experimental researchers.

Bilinear Model

At the initial development stage of nonlinear dynamic analysis technique, the elastic-perfectly plastic hysteretic model was used by many investigators because the model was simple and efficient to use.

The maximum displacement of a single-degree-of-freedom system with the elasto-plastic stiffness was found (8) to be practically the same as that for an elastic system having the same period of vibration as long as the system has a period longer than 0.5 sec.

A reinforced concrete or steel member normally exhibits a strain hardening characteristic after yielding. Therefore, a finite positive slope was assigned to the post-yield stiffness. The bilinear model does not dissipate hysteretic energy until yielding is developed.

The hysteresis of a bilinear model is compared with an observed hysteresis obtained from a cantilever reinforced concrete member test (4) in Fig. 6. The bilinear model does not represent the degrading of loading and unloading stiffnesses with increasing displacement amplitude reversals. Therefore, the usage may not be encouraged in a refined nonlinear analysis of a reinforced concrete structure.

Clough's Degrading Stiffness Model

The first qualitative model for the reinforced concrete was developed by Clough (9), who incorporated the stiffness degradation in the bilinear model.

After studying the force-deflection history of well designed reinforced concrete under static load reversals, Clough proposed to modify the bilinear model as follows: during loading, the response should always move toward the previous maximum response point on the force-deformation diagram. Unloading slope is always parallel to the initial elastic slope. This small modification was a significant step toward the development of more realistic hysteretic models for the reinforced concrete. The model simulates the flexural behaviour of the reinforced concrete (Fig. 7).

The effect of stiffness degradation on the ductility demand during earthquake was studied through the response analyses of a series of single-degree-of-freedom systems (9). It was concluded that, (a) the degrading stiffness did not cause any significant change in the ductility factors for long period structures (period longer than 0.6 sec.) compared to the bilinear stiffness; on the other hand, (b) the short period structure with degrading stiffness properties has significantly larger ductility requirements than the corresponding elasto-plastic systems; and (c) the response waveform of a degrading stiffness model was distinctly different from that of any ordinary elasto-plastic model.

The model is relatively simple, and has been used extensively in nonlinear analysis research.

Takeda's Degrading Stiffness Model

A hysteresis model, similar to the Clough model, was developed independently by Takeda, Sozen and Nielsen (10). Takeda's model included stiffness changes at flexural cracking and yielding, and strain hardening characteristics. Unloading stiffness was reduced by an exponential function of previous maximum deformation. Takeda also prepared a set of rules for load reversals at a displacement amplitude less than the previous peak amplitudes. These are major improvements from the Clough model.

Failure or extensive damage caused by shear or bond deterioration was not considered in the model. The Takeda model, similar to the Clough model, simulates dominantly flexural behaviour. However, flexural deformation, connection deformation and bar slip of longitudinal reinforcement within the connection were suggested (10) to be considered in defining the backbone curve.

The hysteresis of a Takeda model is compared with an observed hysteresis (4) in Fig. 8. A major characteristic of the observed hysteresis is represented by the Takeda model. The specimen failed by compressive crushing of concrete, which could not be incorporated by the Takeda model.

To test the goodness of the Takeda model, cantilever columns (152 x 152 x 724-mm) tested on the University of Illinois earthquake simulator was analyzed (10). Calculated acceleration waveforms were favourably compared with the observed waveform as shown in Fig. 9.

Takayanagi Model

A pinching action and strength decay are inevitable in a short and deep member due to bar slip and deterioration in shear resistance. In order to model the behaviour of short-deep beams, Takayanagi and Schnobrich (11) introduced a pinching action and strength decay in the Takeda model. Whenever a working hysteresis loop is located in the positive rotation - negative moment range or the negative rotation - positive moment range, the stiffness is reduced (Fig. 10).

After the rotational spring has exceeded the yield moment, a strength decay is introduced in the hysteresis loops on subsequent cycles. The rate of the strength decay is assumed to proportionally increase with rotation, and controlled by a guideline. After moment reaches the guideline, the hysteresis curve becomes parallel to the post yield slope of the original primary curve.

The values of guideline for strength decay and pinching stiffness were not related to the member geometry and material properties in their modelling, but rather obtained from load reversal tests of the member. Further research should be encouraged to develop better and rational methods of determining the parameters for pinching and strength-decay properties of the reinforced concrete under load reversals.

Degrading Tri-Linear Hysteresis Model

A model that simulates dominantly flexural stiffness characteristics of the reinforced concrete was developed and used extensively in Japan (12). The backbone curve is a trilinear shape with stiffness changes at flexural cracking and yielding. Up to yielding, the model behaves in the same way as the bilinear model. Once deformation exceeds the yield point, the model behaves as a perfectly plastic system. Upon unloading, the unloading point is treated as a new "yield" point, and unloading stiffnesses corresponding to pre- and post-cracking are reduced proportionately so that the behaviour becomes the same as the bilinear model in a range between the positive and negative "yield" points.

This model is extremely similar, in nature, to the bilinear model, and is simple to use in an analysis. The hysteresis of a degrading bilinear model is compared with the observed hysteresis (4) in Fig. 11. Although the degrading trilinear model includes a cracking point in the hysteresis, the cracking point of this model should be used to control the fatness of a hysteresis loop (4) rather than to represent an actual flexural cracking point. The model can simulate major flexural characteristics of the reinforced concrete.

ANALYTICAL MODEL FOR REINFORCED CONCRETE MEMBERS

Inelastic deformation of a reinforced concrete member does not concentrate in a critical location, but rather spreads along the member as shown in Fig. 12. Various models have been proposed and used to represent the distribution of stiffness within a reinforced concrete member. This section reviews some representative member models. Different hysteretic models can be assigned to the deformable part of a member model.

The effect of gravity load on the beam behaviour and the contribution of slabs to the structural stiffness will not be discussed.

One-Component Model

An elasto-plastic frame structure can be analyzed by placing a rigid-plastic spring at the location where yielding is expected. The part of a member between the two rigid-plastic springs remains perfectly elastic. All inelastic deformation is assumed to occur in these springs.

This one-component model was generalized by Giberson (13) and used in the nonlinear analysis of frame structures. The bilinear moment-rotation relationship was used for the springs.

Let us consider a simply supported beam subjected to external moments at supports. The member consists of a linearly elastic prismatic member and two rotational springs at its ends, as shown in Fig. 13. Using the fact that moment at an end of the elastic member is the same as that of the spring, and that rotation is the sum of rotations of the spring and of the elastic beam end, a flexibility relation can be formulated.

The major advantage of using the one-component model is that the amount of inelastic deformation and stiffness of a spring depends solely on moment acting in the spring, and is independent of moment acting at the other end. Therefore, it is simple to assign any complicated hysteretic rules to the spring.

This fact is also a weakness of the model because the member end rotation should be dependent on the curvature distribution along the member, hence dependent on moments at both ends. Let us consider two cases of moment distribution along a member AB, causing yielding at member end A, as shown in Fig. 14. Moment at the other end B is equal to the moment at A-end in case I, and is zero in case II. Corresponding curvature distributions can be of the form shown in the same figure for the two cases, and the inelastic rotations at A-end are given by the shaded areas. Although the moments at A-end are the same, case II can be seen to cause larger inelastic rotation at A-end. Consequently, the evaluation of stiffness characteristics of an equivalent inelastic spring becomes a problem. Normal practice in defining the stiffness of an equivalent inelastic spring is to apply imaginary member-end moments of equal magnitude, causing asymmetric moment distribution along the member with the inflection point at mid-span, and member end rotations are calculated.

This method of evaluating member end rotations is acceptable as long as the point of inflection of any member stays close to the mid-point during the oscillation. Such is not warranted. Furthermore, moment distribution of a beam is not linear because of the existence of gravity loads. Thirdly, inelastic deformation normally penetrates into a member, and it is not rational to lump all inelastic deformations at one point. These are rational criticisms against the model.

It is true that the point of inflection does not locate at mid-span during oscillation. The usage of initial location of inflection point was suggested by Suko and Adams (14). However, once yielding is developed at one member end, moment at the other end must increase to resist higher stress, moving the inflection point toward the centre of the member. At the same time, large concentrated rotation starts to occur near the critical section.

Therefore, it can be expected that the performance of the one-component model might be reasonable if the effect of gravity load on the beam deformation is small, and for a relatively low-rise frame structure, in which the inflection point of a column locates reasonably close to mid-height.

Multi-Component Model

In an effort to analyze frame structures well into inelastic range under earthquake excitation, an interesting model was proposed by Clough, Benuska and Wilson (15). A frame member was divided into two imaginary parallel elements: an elasto-plastic element to represent a yielding phenomenon, and a fully elastic element to represent strain hardening behaviour (Fig. 15). When member end moment reaches the yield level, plastic hinge is placed at the end of the elasto-plastic element. Aoyama and Sugano (16) adapted the two-component model into the multi-component model for a generalized inelastic analysis of reinforced concrete structures. Four parallel beams were used to account for flexural cracking, different yield levels at two member ends and strain hardening.

Let us consider a simply supported beam subjected to member end moments (Fig. 15). Suppose this beam consists of imaginary three parallel components. As the three components are placed parallel, and connected at the two ends, the three components have the same rotations at their ends, and the resultant member end moments are the sum of the moments of the three components. Consequently, a stiffness matrix of such a member can be formulated. It is clear that, using the multi-component model, deformation compatibility of the imaginary components is satisfied only at their ends.

The multi-component model appears to have a merit; rotation at one end of a member depends on moments at both ends of the member. In other words, the distribution of moment along a member can be approximately reflected in the analysis. However, the stiffness of the parallel components must be evaluated under a certain assumed moment distribution. Therefore, the stiffness parameters are valid only under such an assumed moment distribution, and are bound to be approximate when the moment distribution becomes drastically different.

Giberson (13) discussed the advantages and disadvantages of the one-component and the multi-component models, and concluded that the one-component model was more versatile than the multi-component model because the multi-component model was restricted to the bilinear-type hysteresis characteristics. Therefore, using the multi-component model, it is difficult to simulate the fundamental characteristic of the reinforced concrete: the stiffness degradation.

Connected Two-Cantilever Model

When a frame is analyzed under lateral loads only, reflecting the effect of gravity load on beams, member moment distributes linearly, normally having the inflection point within the member. From the similarity of moment distribution, the member can be considered to consist of two imaginary cantilevers, free at the point of contraflexure and fixed at the member end, which are connected at the inflection point satisfying the continuity of displacement and rotation (17).

Force-deflection relationships of cantilever beams can be obtained experimentally through the test of a simply supported member.

The flexibility relation of a member was formulated by assuming, (a) inflection point does not shift during a short time increment; (b) free-end rotation and displacement are proportional to the beam length and the square of the beam length, respectively; and (c) instantaneous stiffness for shear-rotation and shear-displacement curves of a unit length reference cantilever can be defined by hysteretic models.

The weakness of this method is that the member flexibility matrix is a function of the location of inflection point, and is not of symmetric form. This weakness was caused because the matrix is formulated on the basis of current inflection point, which is assumed to remain in that position during a short time interval. The location of inflection point tends to shift rapidly when the sign of a member end moment changes, which causes a numerical problem. Consequently, this method can not be recommended for a general dynamic analysis. However, the method is useful for incremental static load analysis of a structure.

Discrete Element Model

In order to overcome some difficult problems related to nonlinear analysis of reinforced concrete members, a member can be sub-divided into short line segments along the length, and assign to each short segment a nonlinear hysteretic characteristic. The nonlinear stiffness can be assigned within a segment, or at the connection of two adjacent segments.

Wen and Janssen (18) presented a method for dynamic analysis of a plane frame consisting of elasto-plastic segments. Consequently, mass and flexibility of a member were lumped at the connecting points on a tributary basis, except at beam-column joints which were assumed to be rigid. Powell (19, 20) suggested to use a degrading stiffness model for rigid-inelastic connecting springs (Fig. 16.a). Shorter segments were recommended in a region of high moment, and longer segments in a low moment region.

An alternative method is to divide a member into short segments, each segment with uniform flexural rigidity that varies with a stress-history of the segment (Fig. 16.b). It is easy to handle a local concentration of inelastic action of a member by arranging shorter segments at the location of high concentration of inelastic deformation (11).

These methods are useful when more accurate results are required, or in the analysis of walls. More computational effort is required compared to the other simple methods.

Distributed Flexibility Model

Once cracks develop in a member, the stiffness becomes non-uniform along the member length. Instead of dividing a member into short segments, Takizawa (21) developed a model which assumed a prescribed distribution pattern of cross-sectional flexural flexibility along the member length. A parabolic distribution with an elastic flexibility at the member ends was given by a hysteretic model dependent on a stress history. Therefore, the problem is reduced to formulate an instantaneous stiffness matrix of a non-prismatic member, whose flexural flexibility distributes in a parabolic form.

This is an interesting concept in analyzing an inelastic member. However, the parabolic flexibility distribution may not describe actual concentration of deformation at critical sections (normally at member ends) due to flexural yielding and deformation attributable to slippage of longitudinal reinforcement within a beam-column connection. Inclusion of inelastic springs at locations of concentrated deformation in this model may be a useful solution.

RELIABILITY OF ANALYTICAL MODELS

Earthquake simulator tests provide interesting opportunities to examine the goodness of different analytical models in simulating the observed response of small- to medium-scale highly inelastic model structures. This section reviews the reliability of different analytical models in relation to the capability to simulate the observed behaviour.

These test structures were designed to behave dominantly in flexure, being prevented as much as possible from failing in shear or anchorage because shear and anchorage failures are influenced by the scale effect, and because the two types of failure are not desirable in real construction and are avoided in a design process.

Three-Storey One-Bay Frames (I)

Small scale three-storey one-bay reinforced concrete frames (approximately one-sixth scale model) were tested on the University of Illinois Earthquake Simulator (17). The connected two-cantilever model was used. The stiffness properties of individual members were calculated on the bases of the geometry and material properties. The Takeda model was used to represent the force-deflection of each cantilever model. Member end rotation due to bar slip was approximated by a simplified Takeda model, using a bilinear backbone curve. The bilinear Takeda model is very similar to the Clough model, and does not simulate the "pinching" behaviour due to bar slippage. The damping matrix was assumed to be proportional to the instantaneous stiffness matrix.

The model structure was subjected to a base motion simulating El Centro (NS) 1940 accelerogram with a maximum base acceleration of 0.9 g. The first floor displacement was measured to be as much as four times that corresponding to yielding under static lateral loads.

The analytical models with and without viscous damping favourably simulated the large-amplitude oscillations at 1.0 sec., 2.0 sec., and 5 sec. from the beginning of the motion (Fig. 18). The analytical models, however, failed to simulate the medium and low amplitude oscillations.

Note that the frequencies at the medium to low-amplitude oscillations are higher for the analytical model than for the test structure, which indicates that the test structure was more flexible at low stress level than the analytical model. In order to reproduce lower amplitude oscillations of the observed response waveforms, a slip-type hysteresis model (a model which has a very low stiffness at low stress as shown in Fig. 5) need be introduced in the analysis.

Three-Storey One-Bay Frames (II)

Another set of three-storey one-bay small-scale reinforced concrete frame structures (approximately one-sixth scale model) was tested on the University of Illinois Earthquake Simulator (22). The test structure was similar to the last frame structures. The El Centro (NS) 1940 record was simulated with maximum peak acceleration of 1.1 g. The base motion is significantly more intense than a design earthquake motion.

A special purpose computer program SAKE (23) for a nonlinear dynamic analysis of regular rectangular frame structures was used. A member was represented by the one-component model with two inelastic rotational springs at each member end: one for the flexural deformation and the other for the member end rotation due to bar slip. Takeda models with tri-linear and bi-linear backbone curves were assigned to represent moment-rotation hysteresis behaviour of the two inelastic springs. Two types of damping were used in the analysis: (a) a damping matrix proportional to the mass matrix, and (b) a damping matrix proportional to an instantaneous stiffness matrix. The first mode damping factor was 5% of critical at the initial elastic stage.

Observed and calculated third level displacement waveforms were compared in Fig. 19. The comparison is fair for large amplitude oscillations as before. The waveform at low amplitude oscillations was not simulated well. Again in this analysis, a slip-type characteristic was not incorporated in the analysis.

A fair agreement between the computed using the one-component model and the observed may be attributable to the fact that the inflection point tended to be near the mid-point of each member in such a low-rise frame structure. The usage of the one-component model for high-rise frame structures is cautioned.

Two-Storey One-Bay Frame

Two-storey one-bay medium-scale frame structures (approximately one-half scale model) with slabs were tested on the University of California Earthquake Simulator (24, 25). The structure was analyzed using the two-component model, which can not incorporate the stiffness degradation. In an effort to improve the correlation, the elastic stiffness of two-parallel components was degraded as a function of a first-mode-response amplitude history, including previous maximum displacement amplitude and the number of displacement cycles exceeding a specified value. The parameters which controlled the degradation and deterioration mechanism could not be determined on the basis of material properties and the structural geometry. The initial period of vibration of the mathematical model was adjusted to correspond with that measured before the start of the test.

The observed and the calculated second floor displacement waveforms are compared in Fig. 20. Good correlation can be observed for the waveforms with large amplitudes.

Ten-Storey Coupled Shear Walls

Four ten-storey coupled shear walls were tested on the University of Illinois Earthquake Simulator (26). Takayanagi and Schnobrich (11) divided a wall into short segments of uniform stiffness (Fig. 16.b), and used the one-component model (Fig. 13) to represent a connecting beam. It was judged that the usage of two-dimensional plane stress elements for the walls was less desirable because such an approach might cost more computational effort without any compensating increase in accuracy.

In the analysis of a coupled wall, axial force in a wall element changes due to the overturning effect. The moment carrying capacity of a reinforced concrete section is sensitive to the axial load. Therefore, Takayanagi and Schnobrich (11) incorporated this effect in the Takeda model by changing the tri-linear moment-curvature backbone curve as a function of existing axial force in the wall element. The flexural, shear and axial rigidities were assumed to be uniform within a wall segment. Shear rigidity was varied proportional to flexural rigidity. The Takayanagi model with pinching action and strength decay was used in a beam.

The comparison of the measured and calculated displacement and acceleration is excellent, as shown in Fig. 21.

Inelastic actions of the connecting beams played a major role in controlling the structural response since the beam strength controlled the axial forces that developed in the wall, and the wall moment capacity was affected by the changes of these axial forces.

It is necessary to include the effects of inelastic axial rigidity of the wall section, and pinching action and strength decay of the connecting beams to reproduce the maximum displacement response and the elongation of the period. The strength decay has a larger effect on the maximum displacement response and on the elongation of the period than does any pinching action. Some stiffness parameters for the walls and connecting beams were defined on the basis of static tests of connecting beam-wall assemblies.

CONCLUSION

The behaviour of reinforced concrete buildings, especially under earthquake motion, was briefly reviewed. When a structure can be idealized as plane structures, the current state of the art provides useful and reliable analytical methods.

The favourable comparison of the measured and the calculated response waveforms encourages the use of right analytical and hysteretic models. It is desirable in developing a mathematical model that all parameters of the proposed model should be evaluated on the basis of the geometry of a structure and the properties of materials.

However, more research is required to understand the effect of slabs, gravity loads and biaxial ground motion on nonlinear behaviour of a three-dimensional reinforced concrete structure.

LIST OF REFERENCES

1. Jennings, P.C. and Kuroiwa, J.H., "Vibration and Soil-Structure Interaction Tests of a Nine-Story Reinforced Concrete Building", Bulletin, Seismological Society of America, Vol. 58, June 1968, pp. 891-916.

2. Mahin, S.A. and Bertero, V.V., "Rate of Loading Effect on Uncracked and Repaired Reinforced Concrete Members", EERC 72-9, University of California, Berkeley, December 1972.

3. Shiga, T., Ogawa, J., Shibata, A. and Shibuya, J., "The Dynamic Properties of Reinforced Concrete Frames", Proceedings, U.S. - Japan Seminar on Earthquake Engineering, Sendai, September 1970, pp. 346-363.

4. Otani, S., Cheung, V. W.-T. and Lai, S.S., "Behaviour of Reinforced Concrete Columns Under Simulated Biaxial Earthquake Loads", to be published in Proceedings, International Symposium on Behaviour of Building Systems and Building Components, Vanderbilt University, Nashville, March 1979.

5. Celebi, M. and Penzien, J., "Experimental Investigation into the Seismic Behaviour of Critical Region of Reinforced Concrete Components as Influenced by Moment and Shear", EERC 73-4, University of California, Berkeley, January 1973.

6. Wight, J.K. and Sozen, M.A., "Shear Strength Decay in Reinforced Concrete Columns Subjected to Large Deflection Reversals", SRS No. 403, University of Illinois, Urbana, August 1973.

7. Bertero, V.V. and Popov, E.P., "Seismic Behaviour of Moment-Resisting Reinforced Concrete Frames", Publication SP-53, Reinforced Concrete Structures in Seismic Zones, ACI, 1977, pp. 247-292.

8. Veletsos, A.S. and Newmark, N.M., "Effect of Inelastic Behaviour on the Response of Simple Systems to Earthquake Motions", Proceedings, Second World Conference on Earthquake Engineering, Tokyo and Kyoto, Vol. II, pp. 895-912.

9. Clough, R.W., "Effect of Stiffness Degradation on Earthquake Ductility Requirements", Report 66-16, Structures and Materials Research, Structural Engineering Laboratory, University of California, Berkeley, October 1966.

10. Takeda, T., Sozen, M.A. and Nielsen, N.N., "Reinforced Concrete Response to Simulated Earthquakes", Journal, Structural Division, ASCE, Vol. 96, December 1970, pp. 2557-73.

11. Takayanagi, T. and Schnobrich, W.C., "Computed Behaviour of Reinforced Concrete Coupled Shear Walls", SRS No. 434, University of Illinois, Urbana, December 1976.

12. Fukada, Y., "Study on the Restoring Force Characteristics of Reinforced Concrete Buildings" (in Japanese), Proceedings, Kanto District Symposium, Architectural Institute of Japan, No. 40, 1969.

13. Giberson, M.F., "The Response of Nonlinear Multi-Story Structures Subjected to Earthquake Excitation", Report, EERL, California Institute of Technology, Pasadena, May 1967.

14. Suko, M. and Adams, P.F., "Dynamic Analysis of Multibay Multi-Story Frames", Journal, Structural Division, ASCE, Vol. 97, October 1971, pp. 2519-2533.

15. Clough, R.W., Bernuska, K.L. and Wilson, E.L., "Inelastic Earthquake Response of Tall Buildings", Proceedings, Third World Conference on Earthquake Engineering, New Zealand, January 1965, Vol. II, Session II, pp. 68-89.

16. Aoyama, H. and Sugano, T., "A Generalized Inelastic Analysis of Reinforced Concrete Structures Based on the Tests of Members", Recent Researches of Structural Mechanics - Contribution in Honour of the 60th Birthday of Professor Y. Tsuboi, Uno-Shoten, Tokyo, pp. 15-30, 1968.

17. Otani, S. and Sozen, M.A., "Behaviour of Multi-Story Reinforced Concrete Frames During Earthquakes", SRS No. 392, University of Illinois, Urbana, November 1972.

18. Wen, R.D. and Janssen, J.G., "Dynamic Analysis of Elasto-Inelastic Frames", Third World Conference on Earthquake Engineering, Proceedings, Third World Conference on Earthquake Engineering, New Zealand, January 1965, Vol. II, pp. 713-729.

19. Kanaan, A.E. and Powell, G.H., "DRAIN-2D, A General Purpose Computer Program for Dynamic Analysis of Inelastic Plane Structures", EERC 73-6, University of California, Berkeley, April 1973.

20. Powell, G.H., "Supplement to Computer Program DRAIN-2D, Supplement to Report", "DRAIN-2D USER's GUIDE", University of California, Berkeley, August 1975.

21. Takizawa, H., "Strong Motion Response Analysis of Reinforced Concrete Buildings" (in Japanese), Concrete Journal, Japan National Council on Concrete, Vol. II, No. 2, February 1973, pp. 10-21.

22. Otani, S., "Earthquake Tests of Shear Wall-Frame Structures to Failure", Proceedings, ASCE/EMD Special Conference, Dynamic Response of Structures, University of California, Los Angeles, March 1976, pp. 298-307.

23. Otani, S., "SAKE - A Computer Program for Inelastic Response of R/C Frames to Earthquakes", SRS No. 413, University of Illinois, Urbana, November 1974.

24. Hidalgo, P. and Clough, R.W., "Earthquake Simulator Study of a Reinforced Concrete Frame", EERC 74-13, University of California, Berkeley, December 1974.

25. Clough, R.W. and Gidwani, J., "Reinforced Concrete Frame 2: Seismic Testing and Analytical Correlation", EERC 76-15, University of California, Berkeley, June 1976.

26. Aristizabal-Ochoa, J.D. and Sozen, M.A., "Behaviour of Ten-Story Reinforced Concrete Walls Subjected to Earthquake Motion", SRS No. 431, University of Illinois, Urbana, October 1976.

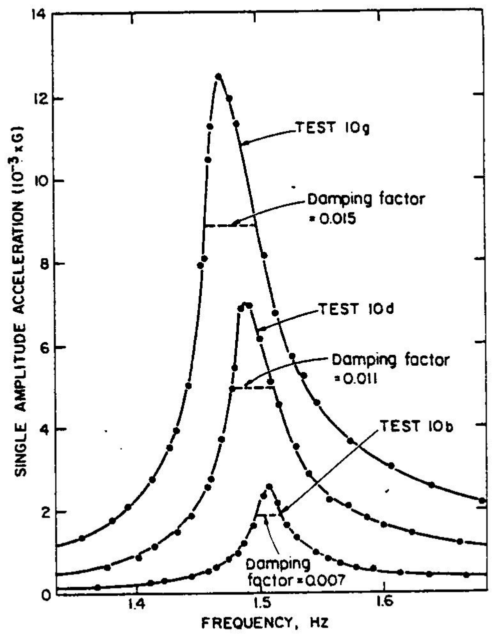

Fig. 1: Observed Acceleration Amplitudes from Steady-State Test (1)

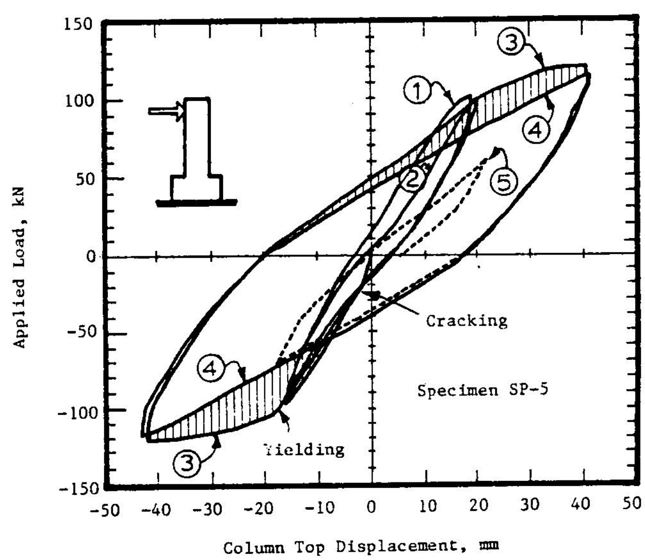

Fig. 2: Hysteretic Characteristics of Reinforced Concrete Member (4)

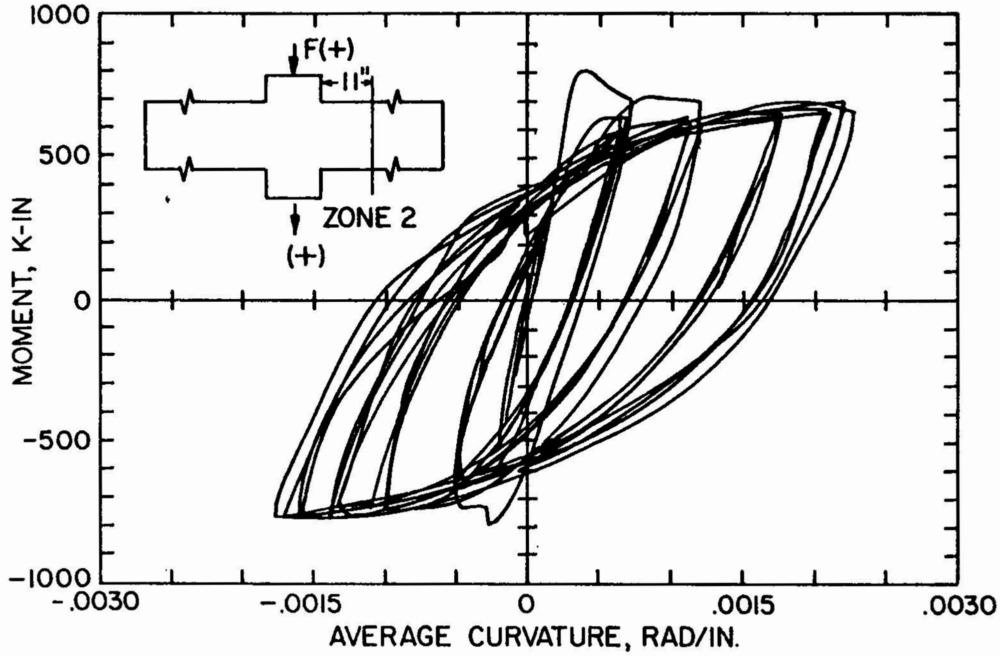

Fig. 3: Flexural Deformation Characteristics (5)

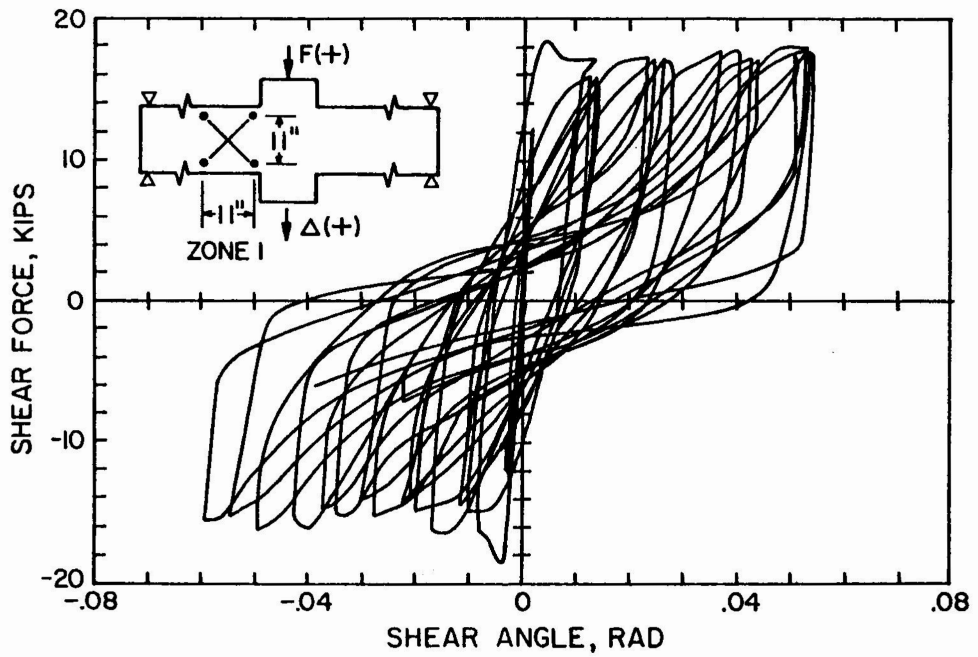

Fig. 4: Shear Deformation Characteristics (5)

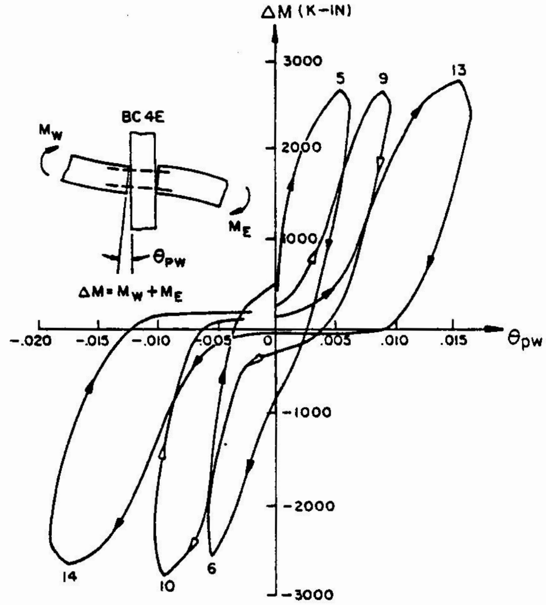

Fig. 5: Rotation Due to Bar Slip (7)

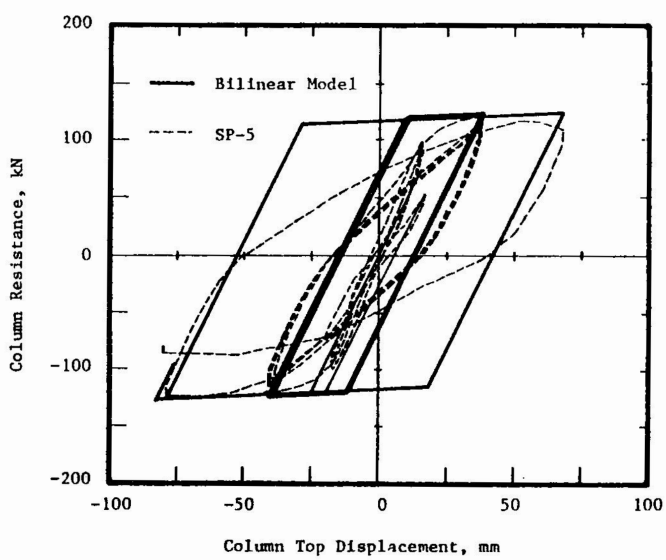

Fig. 6: Bilinear Hysteresis Model

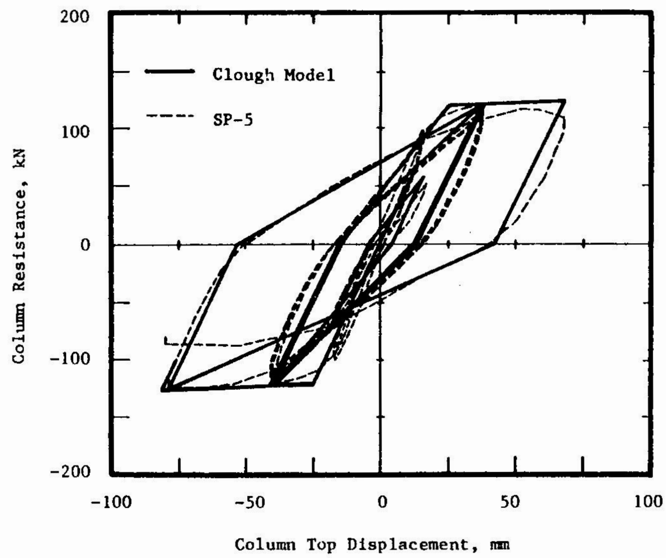

Fig. 7: Clough's Degrading Stiffness Model

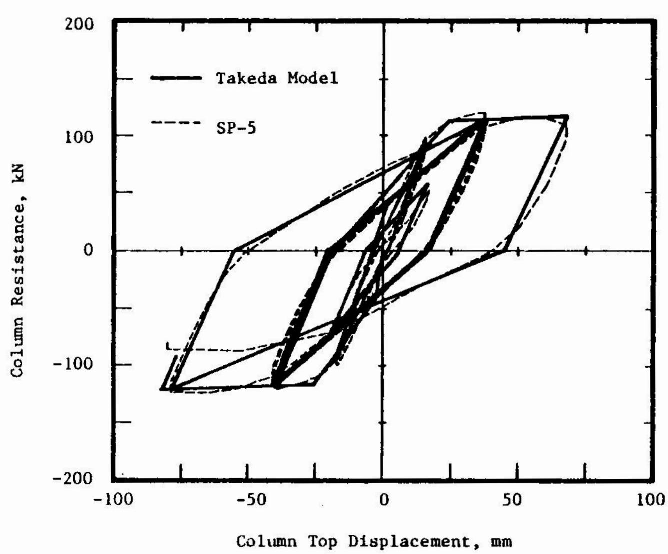

Fig. 8: Takeda's Degrading Stiffness Model

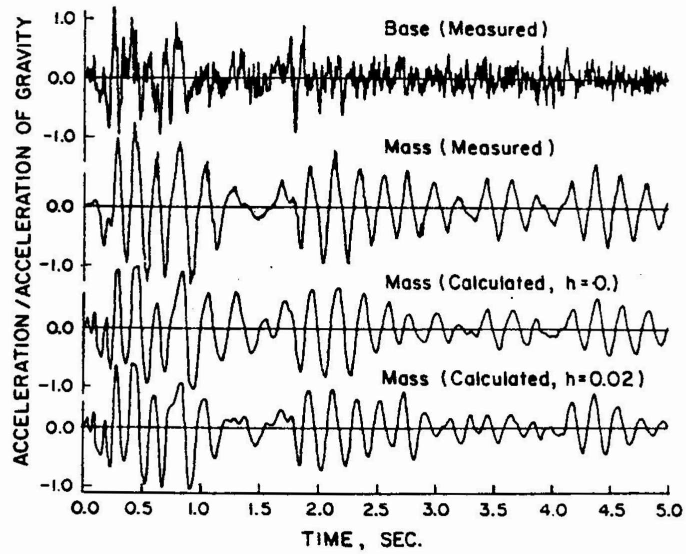

Fig. 9: Takeda Model Applied to Cantilever R/C Column Analysis (10)

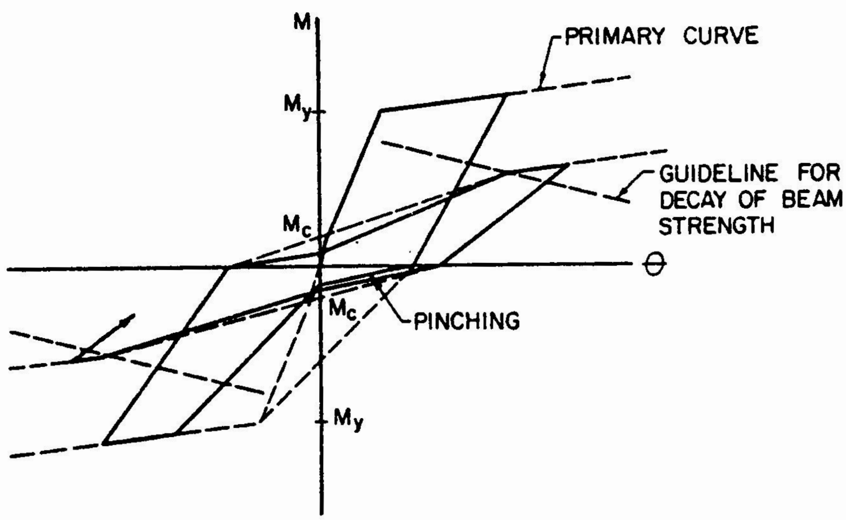

Fig. 10: Takayanagi Model with Pinching and Strength Decay (11)

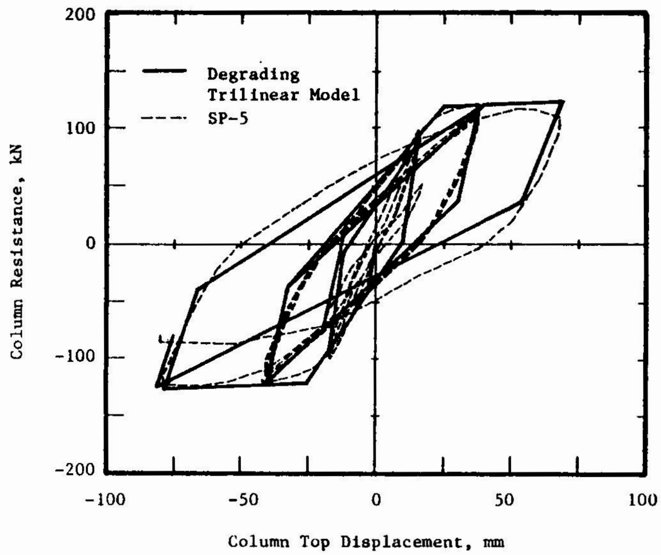

Fig. 11: Degrading Trilinear Model

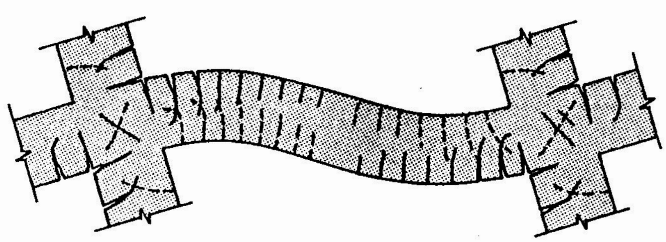

Fig. 12: Deformation of Beam Under Gravity and Earthquake Loads

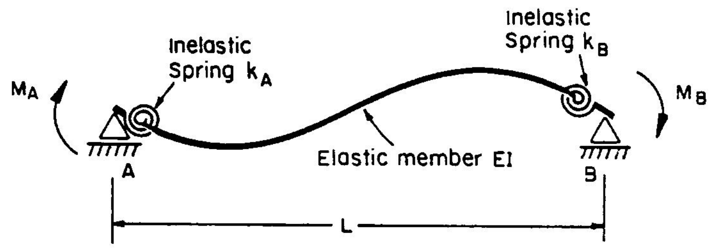

Fig. 13: One-Component Model

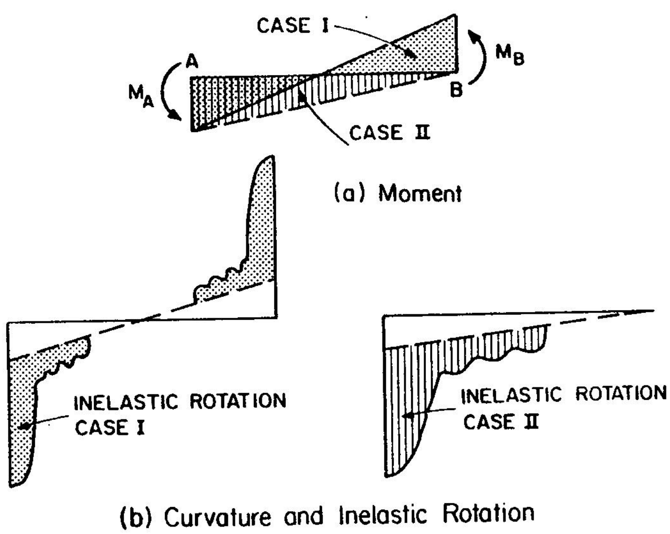

Fig. 14: Inelastic Rotation of Beam

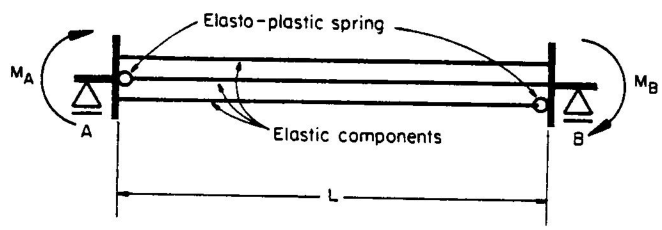

Fig. 15: Multi-Component Model

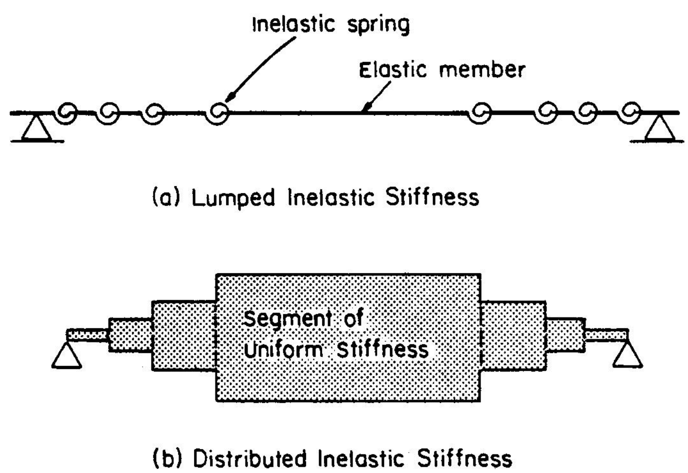

Fig. 16: Discrete Element Model

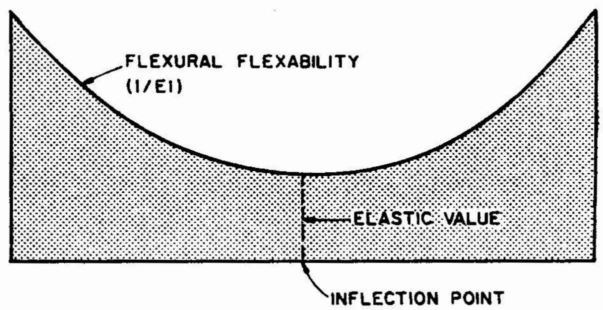

Fig. 17: Distributed Flexibility Model

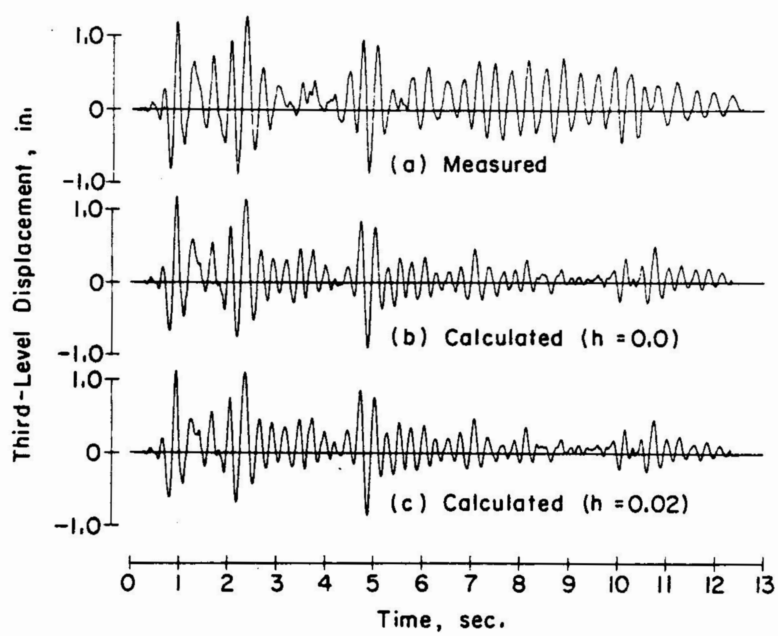

Fig. 18: Connected Two-Cantilever Model Applied to Three-Storey Frame Analysis (17)

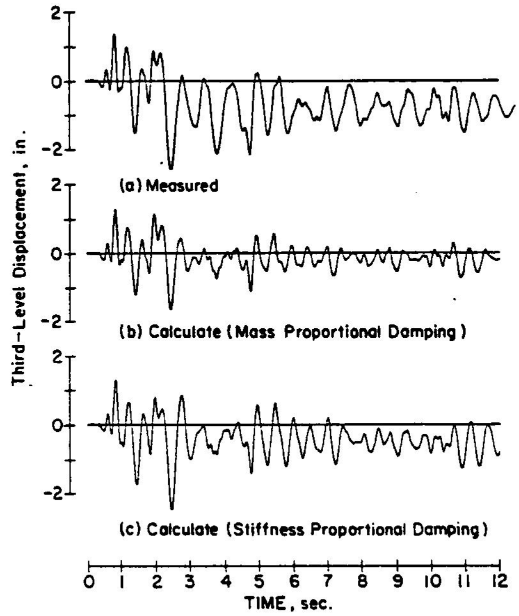

Fig. 19: One-Component Model Applied to Three-Storey Frame Analysis (22)

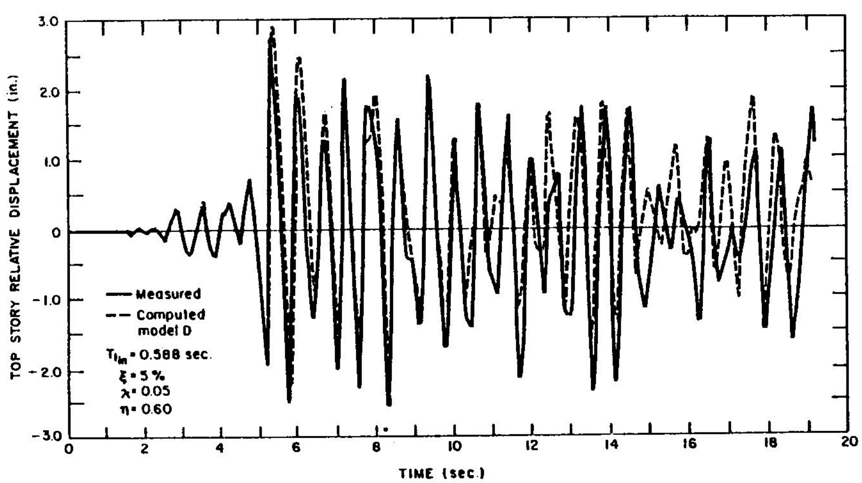

Fig. 20: Two-Component Model Applied to Two-Storey Frame Analysis (24)

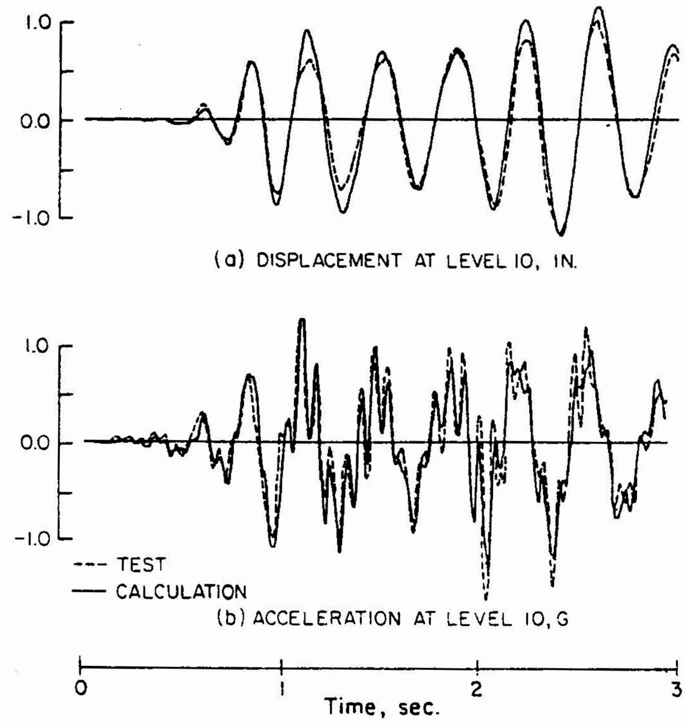

Fig. 21: Analysis of Ten-Storey Coupled Shear Wall (11)